$= 9.274 \times 10^{-24}\ Am^2$

(1.1) $\vec{\mu} = \sqrt{3}\, g\mu_0/2 = \sqrt{3}\mu_B$

(1.2) $\hat{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\hat{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\hat{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\hat{S}^2 |\Psi\rangle = \frac{1}{4}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$= \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \ldots$ ✓

$[\hat{S}_x, \hat{S}_y] = i\hat{S}_z = \hat{S}_x\hat{S}_y - \hat{S}_y\hat{S}_x$

$= \frac{1}{4}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} - \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} + \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$

$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$

$= \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} = \frac{i}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \boxed{iS_z}$

of these 68,

98 - 104 excluded already

up to 90 (+ including) 30-32

32 - 30 + 1 = 3

90 - 30 + 1 = 61

60 in length

up to inc. 84

84 - 30 + 1 = 55

no sections, a^*, step ~~2~~ 16 ↑, zoom x9, left x3,

a^*

a^*, up 9, in 10, left 2, iso 0.000670

0.0008952

(1.3

$[\hat{S}^2, \hat{S}_z] = 0 \; ; \; \hat{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2 = \frac{3}{4}$

$$\hookrightarrow \frac{3}{4}\left(\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right) - \left(\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right)\frac{3}{4} = 0$$

$\hat{S}_+ = \hat{S}_x + i\hat{S}_y \Rightarrow [\hat{S}_+, \hat{S}_-] = (\hat{S}_x + i\hat{S}_y)(\hat{S}_x - i\hat{S}_y) - (\hat{S}_x - i\hat{S}_y)(\hat{S}_x + i\hat{S}_y)$

$\hat{S}_- = \hat{S}_x - i\hat{S}_y$

$$= \cancel{\hat{S}_x^2} - \hat{S}_x(i\hat{S}_y) + i\hat{S}_y\hat{S}_x - \cancel{i^2\hat{S}_y^2}$$
$$- \cancel{\hat{S}_x^2} - \hat{S}_x(i\hat{S}_y) + (i\hat{S}_y)\hat{S}_x + \cancel{i\hat{S}_y(i\hat{S}_y)}$$

$$= -\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix}0&-i^2\\i^2&0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0&-i^2\\i^2&0\end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix} - \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\cdot\frac{1}{2}\begin{pmatrix}0&-i^2\\i^2&0\end{pmatrix}$$

$$= -\frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix} + \frac{1}{4}\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} \qquad + \frac{1}{2}\begin{pmatrix}0&-i^2\\i^2&0\end{pmatrix}\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$- \frac{1}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix} + \frac{1}{4}\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$= -\frac{1}{4}\begin{pmatrix}-1&0\\0&1\end{pmatrix} + \frac{1}{4}\begin{pmatrix}0&0\\0&-1\end{pmatrix} - \frac{1}{4}\begin{pmatrix}-1&0\\0&0\end{pmatrix} + \frac{1}{4}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$= \frac{1}{4}\left[\begin{pmatrix}1&0\\0&-1\end{pmatrix} + \begin{pmatrix}0&0\\0&-1\end{pmatrix} + \begin{pmatrix}1&0\\0&0\end{pmatrix} + \begin{pmatrix}1&0\\0&-1\end{pmatrix}\right]$$

$$= \frac{1}{4}\left[\begin{pmatrix}3&0\\0&-3\end{pmatrix}\right] = \frac{3}{4}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \frac{3}{2}\cdot\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \frac{3}{2}\hat{S}_z$$

$\hat{S}_+ = \hat{S}_x + i\hat{S}_y$ Prove: $[\hat{S}_+, \hat{S}_-] = 2\hat{S}_z$

$\hat{S}_- = \hat{S}_x - i\hat{S}_y$ $\qquad \hat{S}_+\hat{S}_- + \hat{S}_-\hat{S}_+ = 2(\hat{S}_x^2 + \hat{S}_y^2)$

$$[\hat{S}_+, \hat{S}_-] = (\hat{S}_x + i\hat{S}_y)(\hat{S}_x - i\hat{S}_y) - (\hat{S}_x - i\hat{S}_y)(\hat{S}_x + i\hat{S}_y)$$
$$= \cancel{\hat{S}_x^2} - i\hat{S}_x\hat{S}_y + i\hat{S}_y\hat{S}_x + \cancel{\hat{S}_y^2} - \cancel{\hat{S}_x^2} - i\hat{S}_x\hat{S}_y + i\hat{S}_y\hat{S}_x - \cancel{\hat{S}_y^2}$$
$$= 2i(\hat{S}_y\hat{S}_x - \hat{S}_x\hat{S}_y) = -2i[\hat{S}_x, \hat{S}_y] = -2i(i\hat{S}_z) = \boxed{2\hat{S}_z}$$

$$\hat{S}_+\hat{S}_- + \hat{S}_-\hat{S}_+ = \hat{S}_x^2 \cancel{- i\hat{S}_x\hat{S}_y} \cancel{+ i\hat{S}_y\hat{S}_x} + \hat{S}_y^2 + \hat{S}_x^2 \cancel{+ i\hat{S}_x\hat{S}_y} \cancel{- i\hat{S}_y\hat{S}_x} + S_y^2$$
$$\boxed{= 2(\hat{S}_x^2 + S_y^2)}$$

(1.4) $[\hat S_x, \hat S_y] = i\hat S_z$ Prove: $[\hat S\cdot\hat X, \hat S] = i\hat S\times\hat X$

$$[\hat S\cdot\vec X, \hat S\cdot\vec Y] = (\hat S\cdot\vec X)(\hat S\cdot\vec Y) - (\hat S\cdot\vec Y)(\hat S\cdot\vec X)$$

$$= \left(\tfrac12\hat\sigma\cdot\hat X\right)\left(\tfrac12\hat\sigma\cdot\vec Y\right) - \left(\tfrac12\hat\sigma\cdot\vec Y\right)\left(\tfrac12\hat\sigma\cdot\vec X\right)$$

$$= \frac14\left[(\hat\sigma\cdot\vec X)(\hat\sigma\cdot\vec Y) - (\hat\sigma\cdot\vec Y)(\hat\sigma\cdot\vec X)\right]$$

$$= \frac14\left[\left(\cancel{\vec X\cdot\vec Y} + i\hat\sigma\cdot(\vec X\times\vec Y)\right) - \left(\cancel{\vec Y\cdot\vec X} + i\hat\sigma\cdot(\vec Y\times\vec X)\right)\right]$$

$$= \frac i4\left[\hat\sigma\cdot(\vec X\times\vec Y) - \hat\sigma\cdot(\vec Y\times\vec X)\right] = \frac i2\hat\sigma\cdot(\vec X\times\vec Y)$$

$$= i\hat S\cdot(\vec X\times\vec Y)$$

$$= i(\hat S\times\vec X)\cdot\vec Y$$

$$(\hat\sigma\cdot\vec a)(\hat\sigma\cdot\vec b) = \vec a\cdot\vec b + i\hat\sigma\cdot(\vec a\times\vec b)$$

$[\hat S_x, \hat S_y] = i\hat S_z$

$$\begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$(\hat S\cdot\vec X)(\hat S\cdot\vec Y) = \frac14\vec X\cdot\vec Y + \frac i2\hat S\cdot(\vec X\times\vec Y)$$

$$(S_xa_x + S_ya_y + S_za_z)(S_xb_x + S_yb_y + S_zb_z)$$

$$= \frac14(a_xb_x + a_yb_y + a_zb_z)$$
$$+ \frac i2\hat S\cdot\left(\vec i(a_yb_z - a_zb_y) + \vec j(a_xb_z - a_zb_x) + \vec k(a_xb_y - a_yb_x)\right)$$

$$= \frac14(a_xb_x + a_yb_y + a_zb_z)$$
$$+ \frac i2\left(S_x(a_yb_z - a_zb_y) + S_y(a_xb_z - a_zb_x) + S_z(a_xb_y - a_yb_x)\right)$$

$$\left((\sigma_x,\sigma_y,\sigma_z)\begin{pmatrix}a_x\\a_y\\a_z\end{pmatrix}\right)\left((\sigma_x,\sigma_y,\sigma_z)\begin{pmatrix}b_x\\b_y\\b_z\end{pmatrix}\right) = \left((a_x,a_y,a_z)\begin{pmatrix}b_x\\b_y\\b_z\end{pmatrix}\right)$$
$$+ i(\sigma_x,\sigma_y,\sigma_z)\begin{pmatrix}a_yb_z - a_zb_y\\a_xb_z - a_zb_x\\a_xb_y - a_yb_x\end{pmatrix}$$

$$\begin{vmatrix} i & j & k \\ S_x & S_y & S_z \\ a_x & a_y & a_z \end{vmatrix}$$

$$i\sigma_x(a_yb_z - a_zb_y) + i\sigma_y(a_xb_z - a_zb_x) + i\sigma_z(a_xb_y - a_yb_x)$$

$$(\hat S\cdot\vec a)(\hat S\cdot\vec b) = \frac14\vec a\cdot\vec b + \frac i2\hat S\cdot(\vec a\times\vec b)$$
$$(\hat S\cdot\vec a)\hat S = \frac14\vec a + \frac i2(\hat S\times\vec a)$$
$$(\hat S_xa_x + \hat S_ya_y + \hat S_za_z)(\hat S) = \frac14\vec a + \frac i2\vec i(S_ya_z - S_za_y)$$

$$S_x S_y - S_y S_x = i S_z$$

$$\hat{S}_x a_x + \hat{S}_y a_y + \hat{S}_z a_z \begin{pmatrix} \hat{S}_x \\ \hat{S}_y \\ \hat{S}_z \end{pmatrix} = \frac{1}{4}\begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix} + \frac{i}{2}\begin{pmatrix} S_y a_z - S_z a_y \\ S_x a_z - S_z a_x \\ S_x a_y - S_y a_x \end{pmatrix}$$

$$(\hat{S}_x a_x + \hat{S}_y a_y + \hat{S}_z a_z)\hat{S}_x = \frac{1}{4} a_x + \frac{i}{2}(S_y a_z - S_z a_y)$$

$$\hat{S} = \frac{1}{2}\hat{\sigma}$$

$$(\hat{\sigma}\cdot\vec{a})(\hat{\sigma}\cdot\vec{b}) = \vec{a}\cdot\vec{b} + i\hat{\sigma}\cdot(\vec{a}\times\vec{b})$$

$$[\sigma_x, \sigma_y] = i\sigma_z$$
$$[\sigma_x, \sigma_z] = -i\sigma_y$$

to get: $[\hat{S}_x, \hat{S}_y] = i\hat{S}_z$

$$S_x = \frac{1}{2}\sigma_x$$
$$S_y = \frac{1}{2}\sigma_y \qquad \Rightarrow [\hat{S}_x, \hat{S}_y] = \hat{S}_x\hat{S}_y - \hat{S}_y\hat{S}_x$$
$$= \frac{1}{4}(\sigma_x\sigma_y - \sigma_y\sigma_x)$$

$$(\hat{\sigma}\cdot\vec{a})(\hat{\sigma}\cdot\vec{b}) - \vec{a}\cdot\vec{b} = i\hat{\sigma}\cdot(\vec{a}\times\vec{b})$$

$$(\sigma_x a_x + \sigma_y a_y + \sigma_z a_z)(\sigma_x b_x + \sigma_y b_y + \sigma_z b_z) - (a_x b_x + a_y b_y + a_z b_z)$$
$$= i\,\hat{\sigma}\cdot\left[(a_y b_z - b_y a_z)\hat{x} - (a_x b_z - b_x a_z)\hat{y} + (a_y b_z - b_y a_z)\hat{z}\right]$$
$$= i\left[\sigma_x(a_y b_z - b_y a_z) - \sigma_y(a_x b_z - b_x a_z) + \sigma_z(a_y b_z - b_y a_z)\right]$$

$$(\sigma_x a_x + \sigma_y a_y + \sigma_z a_z)(\sigma_x b_x + \sigma_y b_y + \sigma_z b_z) = \sigma_x a_x(\sigma_x b_x + \sigma_y b_y + \sigma_z b_z) + \ldots$$
$$\sigma_y a_y(\sigma_x b_x + \sigma_y b_y + \sigma_z b_z) + \ldots$$
$$\sigma_z a_z(\sigma_x b_x + \sigma_y b_y + \sigma_z b_z)$$

$$= a_x b_x + \sigma_x a_x \sigma_y b_y + \sigma_x a_x \sigma_z b_z + \sigma_y a_y \sigma_x b_x + a_y b_y + \sigma_y a_y \sigma_z b_z + \ldots$$
$$\sigma_z a_z \sigma_x b_x + \sigma_z a_z \sigma_y b_y + a_z b_z$$

$$\Rightarrow \underbrace{\sigma_x\sigma_y}_{i\sigma_z} a_x b_y + \underbrace{\sigma_x\sigma_z}_{-i\sigma_y} a_x b_z + \underbrace{\sigma_y\sigma_x}_{-i\sigma_z} a_y b_x + \underbrace{\sigma_y\sigma_z}_{i\sigma_x} a_y b_z + \underbrace{\sigma_z\sigma_x}_{i\sigma_y} a_z b_y + \underbrace{\sigma_z\sigma_y}_{-i\sigma_x} a_z b_y$$

$$\Rightarrow i\left[\sigma_z a_x b_y - \sigma_y a_y b_z - \sigma_z a_y b_z + \sigma_x a_y b_z + \sigma_y a_z b_y - \sigma_x a_z b_y\right]$$
$$i\left[\sigma_x(a_y b_z - a_z b_y) - \sigma_y(a_x b_z - a_z b_x) + \sigma_z(a_x b_y - a_y b_x)\right]$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[\hat{S}\cdot\vec{X}, \hat{S}\cdot\vec{Y}] = i(\hat{S}\times\vec{X})\cdot\vec{Y}$$
$$[\hat{S}\cdot\vec{X}, \hat{S}] = i(\hat{S}\times\vec{X})$$
$$[\hat{S}\cdot\vec{X}, \hat{S}]\vec{Y} = [\hat{S}\cdot\vec{X}, \hat{S}\cdot\vec{Y}]$$

☆
$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \sigma_x^2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\cdot\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbb{I} = \mathbb{I}$$
$$\sigma_y = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Rightarrow \sigma_y^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\cdot\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbb{I}$$
$$\sigma_z^2 = \mathbb{I}$$

$$\sigma_x\cdot\sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\cdot\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = i\sigma_z$$
$$\sigma_x\cdot\sigma_z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = i\sigma_y$$
$$\sigma_y\cdot\sigma_z = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\cdot\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix} = i\sigma_x$$

$$[\sigma_x, \sigma_y] = i\sigma_z$$

(1.4 again)

$$(\hat{\sigma}\cdot\vec{a})(\hat{\sigma}\cdot\vec{b}) = (\sigma_x a_x + \sigma_y a_y + \sigma_z a_z)(\sigma_x b_x + \sigma_y b_y + \sigma_z b_z)$$
$$= \sigma_x a_x \sigma_x b_x + \sigma_x a_x \sigma_y b_y + \sigma_x a_x \sigma_z b_z$$
$$+ \sigma_y a_y \sigma_x b_x + \sigma_y a_y \sigma_y b_y + \sigma_y a_y \sigma_z b_z$$
$$+ \sigma_z a_z \sigma_x b_x + \sigma_z a_z \sigma_y b_y + \sigma_z a_z \sigma_z b_z$$

$$= \cancelto{1}{\sigma_x^2} a_x b_x + \sigma_x\sigma_y a_x b_y + \sigma_x\sigma_z a_x b_z$$
$$+ \sigma_y\sigma_x a_y b_x + \cancelto{1}{\sigma_y^2} a_y b_y + \sigma_y\sigma_z a_y a_z$$
$$+ \sigma_z\sigma_x a_z b_x + \sigma_z\sigma_y a_z b_y + \cancelto{1}{\sigma_z^2} a_z b_z \qquad \text{Go to ☆}$$

$$= a_x b_x + i\sigma_z a_x b_y + i\sigma_y a_x b_z - i\sigma_z a_y b_x + a_y b_y$$
$$+ i\sigma_x a_y a_z + i\sigma_y a_z b_x - i\sigma_x a_z b_y + a_z b_z$$

$$= a_x b_x + a_y b_y + a_z b_z + i\sigma_x(a_y a_z - a_z b_y)$$
$$+ i\sigma_y(a_x b_z - a_z b_x)$$
$$+ i\sigma_z(a_x b_y - a_y b_x)$$

$$= \vec{a}\cdot\vec{b} + i\hat{\sigma}\cdot(\vec{a}\times\vec{b})$$

So, $(\hat{S}\cdot\vec{X})(\hat{S}\cdot\vec{Y}) = \frac{1}{4}\vec{X}\cdot\vec{Y} + \frac{i}{2}\hat{S}\cdot(\vec{X}\times\vec{Y})$

and $[\hat{S}\cdot\vec{X}, \hat{S}\cdot\vec{Y}] = (\hat{S}\cdot\vec{X})(\hat{S}\cdot\vec{Y}) - (\hat{S}\cdot\vec{Y})(\hat{S}\cdot\vec{X})$

$$= \cancel{\frac{1}{4}\vec{X}\cdot\vec{Y}} + \frac{i}{2}\hat{S}\cdot(\vec{X}\times\vec{Y}) - \cancel{\frac{1}{4}\vec{Y}\cdot\vec{X}} - \frac{i}{2}\hat{S}\cdot(\vec{Y}\times\vec{X})$$

$= \frac{i}{2}\left(\hat{S}\cdot(\vec{X}\times\vec{Y}) - \hat{S}\cdot(\vec{Y}\times\vec{X})\right) = i\hat{S}\cdot(\vec{X}\times\vec{Y})$

$\boxed{= i(\hat{S}\times\vec{X})\cdot\vec{Y} \text{ (end?)}}$

(1.5) $[\hat{S}_+, \hat{S}_-] = 2\hat{S}_z$ (1.58) show $\hat{S}_\pm |S, S_z\rangle =$

$\hat{S}_+\hat{S}_- + \hat{S}_-\hat{S}_+ = 2(\hat{S}_x^2 + \hat{S}_y^2)$ $\qquad \sqrt{S(S+1) - S_z(S_z \pm 1)}\,|S, S_z \pm 1\rangle$

$|S, S_z\rangle$ represents a state w/ total spin ang. mom. $S(S+1)\hbar^2$ and z-component of ang. mom. $S_z\hbar$

Note: $[\hat{S}_z, \hat{S}_\pm] = \pm\hat{S}_\pm$ so $\hat{S}_z\hat{S}_\pm = \pm\hat{S}_\pm + \hat{S}_\pm S_z$

Can write

$$\hat{S}_z(\hat{S}_\pm|S, S_z\rangle) = (\hat{S}_\pm\hat{S}_z \pm \hat{S}_\pm)|S, S_z\rangle$$
$$= (\hat{S}_z \pm 1)(\hat{S}_\pm|S, S_z\rangle)$$

$$\hat{S}_+\hat{S}_- - \hat{S}_-\hat{S}_+ = 2\hat{S}_z$$
$$\hat{S}_+\hat{S}_- + \hat{S}_-\hat{S}_+ = 2(\hat{S}_x^2 + \hat{S}_y^2) = 2(\hat{S}^2 - \hat{S}_z^2)$$

$$\hat{S}_+\hat{S}_- = 2(\hat{S}_x^2 + \hat{S}_y^2) - \hat{S}_-\hat{S}_+ \qquad \hat{S}_-\hat{S}_+ = 2\hat{S}^2 - 2\hat{S}_z^2 - \hat{S}_+\hat{S}_-$$
$$= 2\hat{S}^2 - 2\hat{S}_z^2 + 2\hat{S}_z - \hat{S}_+\hat{S}_- \qquad = 2\hat{S}^2 - 2\hat{S}_z^2 - 2\hat{S}_z - \hat{S}_-\hat{S}_+$$

$$2\hat{S}_+\hat{S}_- = 2\hat{S}^2 - 2\hat{S}_z^2 + 2\hat{S}_z \Rightarrow \hat{S}_+\hat{S}_- = \hat{S}^2 - \hat{S}_z^2 + S_z$$
$$2\hat{S}_-\hat{S}_+ = 2\hat{S}^2 - 2\hat{S}_z^2 - 2\hat{S}_z \Rightarrow \hat{S}_-\hat{S}_+ = \hat{S}^2 - \hat{S}_z^2 - S_z$$

$$\langle S, S_z|\hat{S}_-\hat{S}_+|S, S_z\rangle = S(S+1) - S_z^2 - S_z = S(S+1) - S_z(S_z+1)$$
$$\langle S, S_z|\hat{S}_+\hat{S}_-|S, S_z\rangle = S(S+1) - S_z^2 + S_z = S(S+1) - S_z(S_z-1)$$

(1.6) Uniform $\vec{B}$ is consistent w/ writing $\vec{A} = \frac{1}{2}(\vec{B}\times\vec{r})$

$\vec{B} = (0, 0, B) \Rightarrow \vec{B}\times\vec{r} = \begin{vmatrix} i & j & k \\ 0 & 0 & B \\ x & y & z \end{vmatrix} \Rightarrow (0 - By)\hat{i} - (0 - Bx)\hat{j} + 0$

$\nabla \vec{B}\times\vec{r} = B(-y, -x, 0)$ ✓

$$\begin{vmatrix} p_x+eA_x & p_y+eA_y & p_z+eA_z \\ p_x+eA_x & p_y+eA_y & p_z+eA_z \end{vmatrix} \qquad (p_y p_z - p_z p_y)\hat{i} - (p_x p_z - p_z p_x)\vec{j} + (p_x p_y - p_y p_x)\vec{k}$$

$$(1.7) \quad \hat{K} = \frac{\hat{p}^2}{2m} \; ; \; (\hat{\sigma}\cdot\vec{a})^2 = |\vec{a}|^2$$

$$\frac{(\hat{p}+eA)^2}{2m_e} + g\mu_B B\cdot S$$

$$\hookrightarrow \frac{(\hat{\sigma}\cdot p)^2}{2m_e} \checkmark$$

$$\mu_B = \frac{e\hbar}{2m_e}$$

$$\hat{p} \to \hat{p} + e\hat{A} \quad \text{so} \quad \hat{K} = \frac{(\hat{\sigma}\cdot(\hat{p}+e\hat{A}))^2}{2m_e}$$

$$K = \frac{1}{2m_e}(\hat{\sigma}\cdot(\hat{p}+e\hat{A}))(\hat{\sigma}\cdot(\hat{p}+e\hat{A}))$$

$$\begin{aligned}
\hat{\sigma}\cdot(\hat{p}+e\hat{A}) &= (\hat{\sigma}\cdot\hat{p} + e\hat{\sigma}\cdot\hat{A})(\hat{\sigma}\cdot p + e\vec{\sigma}\cdot\hat{A}) \\
&= (\hat{\sigma}\cdot\hat{p})^2 + 2e(\hat{\sigma}\cdot\hat{p})(\hat{\sigma}\cdot\hat{A}) + e^2(\hat{\sigma}\cdot\hat{A})^2 \\
&= \hat{p}^2 + \cancel{i\sigma\cdot(\hat{p}\times\hat{p})}^{0} + e^2(A^2 + i\sigma\cancel{(A\times A)}^{0}) \\
&= \hat{p}^2 + (eA)^2 + 2e(\hat{\sigma}\cdot\hat{p})(\hat{\sigma}\cdot\hat{A}) \\
&= \hat{p}^2 + (e\hat{A})^2 + 2e(\hat{p}\cdot\hat{A} + i\hat{\sigma}\cdot(\hat{p}\times\hat{A})) \\
&= (\hat{p}^2 + 2e\hat{A}\hat{p} + (e\hat{A})^2) + i\hat{\sigma}\cdot(\hat{p}\times\hat{A}) \\
&= (\hat{p}+e\hat{A})^2 + i\hat{\sigma}\cdot(\hat{p}\times\hat{A})
\end{aligned}$$

$$= \frac{1}{2m_e}\left[(\hat{p}+e\hat{A})^2 + i\hat{\sigma}\cdot((m\vec{v}+q\vec{A})\times\hat{A})\right]$$

$$= \frac{1}{2m_e}\left[(\hat{p}+e\hat{A})^2 + i\hat{\sigma}\cdot[(m\vec{v}\times\hat{A}) + \cancel{(q\vec{A}\times\vec{A})}^{0}]\right]$$

$$= \frac{1}{2m_e}\left[(\hat{p}+e\hat{A})^2 + im\hat{\sigma}\cdot(\vec{v}\times\vec{A})\right]$$

$$= \frac{1}{2m_e}\left[(\hat{p}+e\hat{A})^2 + im\hat{\sigma}\cdot((\nabla\times\hat{r})\times\hat{A})\right]$$

$$= \frac{1}{2m_e}\left[(\hat{p}+e\hat{A})^2 + im\hat{\sigma}\cdot(\hat{r}\times\vec{B})\right]$$

$$= \frac{1}{2m_e}\left[(\hat{p}+e\hat{A})^2 + im\,2\hat{S}\cdot(\hat{r}\times\vec{B})\right]$$

$$= \frac{1}{2m_e}\left[(\hat{p}+e\hat{A})^2 + img\,\hat{B}\cdot\hat{S}\times\vec{r}\right]$$

$$2m_e i\hat{\sigma}\cdot(\hat{\nabla}\times\hat{A}) = \frac{e\hbar}{2m_e}\hat{B}\cdot\hat{\sigma} \quad \ldots$$

$$=$$

$$g\mu_B\hat{B}\cdot\hat{S} = 2\frac{e\hbar}{2m_e}\hat{B}\cdot\left(\frac{1}{2}\hat{\sigma}\right) = \frac{e\hbar}{2m_e}\hat{B}\cdot\hat{\sigma}$$

$$\hat{p} = -i\hbar\nabla$$

$$\mu_B = \frac{e\hbar}{2m_e} \quad \frac{e\hbar}{m_e} = 2\mu_B$$

KE of electron: $\frac{\hat{p}^2}{2m}$ want $\frac{(\sigma\cdot\hat{p})^2}{2m_e}$ use $(\sigma\cdot\vec{a})^2 = |\vec{a}|^2$

$$\hat{\sigma} = \left(\begin{pmatrix}0&1\\1&0\end{pmatrix}, \begin{pmatrix}0&-i\\i&0\end{pmatrix}, \begin{pmatrix}1&0\\0&-1\end{pmatrix}\right)$$

$$\vec{p} = \begin{pmatrix}p_x\\p_y\\p_z\end{pmatrix} \Rightarrow \hat{\sigma}\cdot\vec{p} = \begin{pmatrix}0&1\\1&0\end{pmatrix}p_x + \begin{pmatrix}0&-i\\i&0\end{pmatrix}p_y + \begin{pmatrix}1&0\\0&-1\end{pmatrix}p_z$$

$$= \begin{pmatrix}p_z & p_x - ip_y\\ p_x+ip_y & -p_z\end{pmatrix}$$

$$(\hat{\sigma}\cdot\vec{p})^2 = \begin{pmatrix}p_z & p_x-ip_y\\ p_x+ip_y & -p_z\end{pmatrix}\begin{pmatrix}p_z & p_x-ip_y\\ p_x+ip_y & -p_z\end{pmatrix}$$

$$= \begin{pmatrix}p_z^2 + (p_x-ip_y)(p_x+ip_y) & \cancel{p_z(p_x-ip_y)} - \cancel{p_z(p_x-ip_y)}\\ \cancel{p_z(p_x+ip_y)} - \cancel{p_z(p_x+ip_y)} & (p_x+ip_y)(p_x-ip_y) + p_z^2\end{pmatrix}$$

$$= \begin{pmatrix}p_z^2+p_x^2+p_y^2 & 0\\ 0 & p_z^2+p_x^2+p_y^2\end{pmatrix} = |\vec{p}|^2\begin{pmatrix}1&0\\0&1\end{pmatrix} = \hat{p}^2$$

$$(\hat{\sigma}\cdot(\hat{p}+e\hat{A}))(\hat{\sigma}\cdot(\hat{p}+e\hat{A})) = (\hat{\sigma}\cdot\hat{p} + e\hat{\sigma}\cdot\hat{A})(\hat{\sigma}\cdot\hat{p}+e\hat{\sigma}\cdot\hat{A})$$

$$= (\hat{\sigma}\cdot\hat{p})^2 + 2e(\hat{\sigma}\cdot\hat{p})(\hat{\sigma}\cdot\hat{A}) + e^2(\hat{\sigma}\cdot\hat{A})^2$$

$$= (\hat{\sigma}\cdot\hat{p})^2 + 2e(\hat{p}\cdot\hat{A} + i\hat{\sigma}\cdot(\hat{p}\times\hat{A})) + e^2(\hat{\sigma}\cdot\hat{A})^2$$

$$= \hat{p}^2 + 2e\,\hat{p}\cdot\hat{A} + e\hat{A}^2 + 2ei\,\hat{\sigma}\cdot(\hat{p}\times\hat{A})$$

$$= (\hat{p}+e\hat{A})^2 + 2ei\hat{\sigma}\cdot(\hat{p}\times\hat{A}) = (\hat{p}+e\hat{A})^2 + 2e\hbar\,\hat{\sigma}\cdot(\nabla\times\hat{A})$$

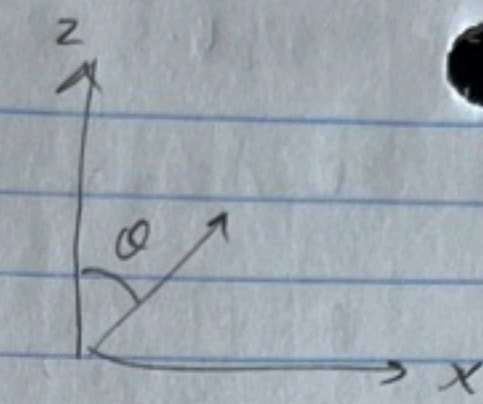
$$= (\hat{p}+e\hat{A})^2 + 2e\hbar\,\hat{\sigma}\cdot\hat{B} = (\hat{p}+e\hat{A})^2 + 4e\hbar\,\hat{S}\cdot\hat{B}$$

so $(\hat{\sigma}\cdot(\hat{p}+e\hat{A}))^2/2m_e = \frac{(\hat{p}+e\hat{A})^2}{2m_e} + \frac{ge\hbar\,\hat{S}\cdot\hat{B}}{m_e} = \frac{(\hat{p}+e\hat{A})^2}{2m_e} + 2g\mu_B\hat{B}\cdot\hat{S}$

and $\boxed{\hat{K} = \frac{(\hat{p}+e\hat{A})^2}{2m_e} + 2g\mu_B\hat{B}\cdot\hat{S}}$

$(M\times N)(N\times 1)$
$= M\times 1$

$\hat{S}_\theta |\uparrow_z\rangle = (\quad) |\uparrow_z\rangle$



(1.8) Zero orb. ang mom. ($l(l+1) = 0$?)

$m_s = +\frac{1}{2}$; $|\uparrow_z\rangle$ state $= \begin{pmatrix}1\\0\end{pmatrix}$

$\hat{S}_\theta = \cos\theta\, \hat{S}_z + \sin\theta\, \hat{S}_x$

$\hat{S}_\theta |\uparrow_z\rangle = 2 |\uparrow_z\rangle$ $\quad \hat{S}_z = \frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ $\quad |\uparrow_z\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$\hat{S}_x = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}$

$$\left(\frac{\cos\theta}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix} + \frac{\sin\theta}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\right)\begin{pmatrix}1\\0\end{pmatrix} = q\begin{pmatrix}1\\0\end{pmatrix}$$

$\theta = 0$

$$\left.\begin{aligned}\frac{\cos\theta}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} &= \frac{\cos\theta}{2}\begin{pmatrix}1\\0\end{pmatrix}\\ \frac{\sin\theta}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} &= \frac{\sin\theta}{2}\begin{pmatrix}0\\1\end{pmatrix}\end{aligned}\right\} \text{LHS} = \frac{\cos\theta}{2}\begin{pmatrix}1\\0\end{pmatrix} + \cancel{\frac{\sin\theta}{2}}\begin{pmatrix}0\\1\end{pmatrix}$$

$$\frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix}$$

$\det(A - \lambda I) = 0$

$$A - \lambda I = \begin{pmatrix}\cos\theta/2 & \sin\theta/2\\ \sin\theta/2 & -\cos\theta/2\end{pmatrix} - \lambda\begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$$\det\begin{pmatrix}\cos\theta/2 - \lambda & \sin\theta/2\\ \sin\theta/2 & -\cos\theta/2 - \lambda\end{pmatrix} = \left(\frac{\cos\theta}{2} - \lambda\right)\left(-\frac{\cos\theta}{2} - \lambda\right)$$

$$= -\frac{\cos^2\theta}{4} - \cancel{\frac{\lambda\cos\theta}{2}} + \cancel{\frac{\lambda\cos\theta}{2}} + \lambda^2$$

$$\begin{pmatrix} & \\ & \end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix} = \frac{1}{2}\begin{pmatrix}x_1\\x_2\end{pmatrix}$$

$$(\sin\theta/2)^2 = \frac{\sin^2\theta}{4}$$

$$= \frac{\sin^2\theta}{4} - \left(\frac{\sin\theta}{2}\right)^2 + \lambda^2$$

$$-\frac{\cos^2\theta}{4} + \lambda^2 - \left(\frac{\sin^2\theta}{4}\right)$$

$$\lambda^2 = \frac{1}{4}(\cos^2\theta + \sin^2\theta) \Rightarrow \lambda^2 = \frac{1}{4}\ ;\ \lambda = \frac{1}{2}, -\frac{1}{2}$$

$$\hat{S}_\theta = \cos\theta\, \hat{S}_z + \sin\theta\, \hat{S}_x$$

$$\hat{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad \hat{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \hat{S}_\theta = \frac{1}{2}\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta/2 - \lambda & \sin\theta/2 \\ \sin\theta/2 & -\frac{\cos\theta}{2} - \lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$= a\begin{pmatrix} \cos\theta/2 - \lambda \\ \sin\theta/2 \end{pmatrix} + b\begin{pmatrix} \sin\theta/2 \\ -\cos\theta/2 - \lambda \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} = \begin{pmatrix} 1/2 \\ 1/2 \end{pmatrix}$$

$$\begin{pmatrix} a(\cos\theta/2) + b(\sin\theta/2) \\ a(\sin\theta/2) + b(-\cos\theta/2) \end{pmatrix} \begin{matrix} = \frac{1}{2} \\ = \frac{1}{2} \end{matrix}$$

$$\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1\cos\theta + x_2\sin\theta \\ x_1\sin\theta - x_2\cos\theta \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$x_1\cos\theta + x_2\sin\theta = x_1 \Rightarrow x_1(1-\cos\theta) = x_2\sin\theta$$
$$x_1\sin\theta - x_2\cos\theta = x_2 \Rightarrow x_2(1+\cos\theta) = x_1\sin\theta$$

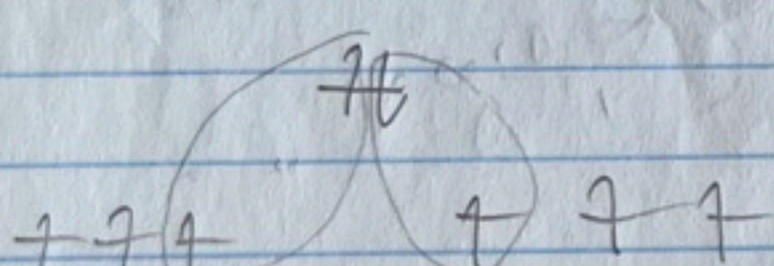
$$\frac{x_1(1-\cos\theta)}{\sin\theta}(1+\cos\theta) = x_1\sin\theta$$

$$x_1\left(\frac{(1-\cos\theta)}{\sin\theta}(1+\cos\theta) - \sin\theta\right) = \emptyset$$

$$\frac{1}{2}\begin{pmatrix} \cos\theta - \frac{1}{2} & \sin\theta \\ \sin\theta & -\cos\theta - \frac{1}{2} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \emptyset \\ \emptyset \end{pmatrix}$$

$$x_1(\cos\theta - \tfrac{1}{2}) + x_2\sin\theta = \emptyset \Rightarrow x_2 = -x_1(\cos\theta - \tfrac{1}{2})/\sin\theta$$
$$x_1(\sin\theta) - x_2(\cos\theta + \tfrac{1}{2}) = \emptyset \qquad x_1 = x_2(\cos\theta + \tfrac{1}{2})/\sin\theta$$

$$x_1\sin\theta + [x_1(\cos\theta - \tfrac{1}{2})]/\sin\theta = \emptyset$$
$$x_1\sin^2\theta + x_1(\cos\theta - \tfrac{1}{2}) = \emptyset$$
$$x_1(\sin^2\theta + \cos\theta - \tfrac{1}{2}) = \emptyset$$



AFM order: monoclinic (H +)

$1 = \cos^2\theta + \sin^2\theta$

$$\left(\frac{\cos\theta}{2} - 1\right)\left(\frac{\cos\theta}{2} + 1\right) = \frac{\cos^2\theta}{4} - 1$$

$$\hat{S}_\theta = \cos\theta\, \hat{S}_z + \sin\theta\, \hat{S}_x = \cos\theta\left(\frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right) + \sin\theta\left(\frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\right)$$

$$(\hat{S}_\theta - I)\vec{v} = \begin{pmatrix}\cos\theta/2 & 0\\ 0 & -\frac{\cos\theta}{2}\end{pmatrix} + \begin{pmatrix}0 & \sin\theta/2\\ \sin\theta/2 & 0\end{pmatrix} - \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$= \begin{pmatrix}\cos\theta/2 - 1 & \sin\theta/2\\ \sin\theta/2 & -\cos\theta/2 - 1\end{pmatrix}\begin{pmatrix}v_1\\ v_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

$$\begin{cases} v_1(\cos\theta/2 - 1) + v_2 \sin\theta/2 = 0 \\ v_1(\sin\theta/2) - v_2(\cos\theta/2 + 1) = 0 \end{cases}$$

$$v_2 = \frac{-2v_1}{\sin\theta}\left(\frac{\cos\theta}{2} - 1\right)$$

$$v_1\left(\frac{\sin\theta}{2}\right) = -\frac{2v_1}{\sin\theta}\left(\frac{\cos\theta}{2} - 1\right)\left(\frac{\cos\theta}{2} + 1\right)$$

$$v_1 = -\frac{4v_1}{\sin^2\theta}\left(\frac{\cos\theta}{2} - 1\right)\left(\frac{\cos\theta}{2} + 1\right)$$

$$= -\frac{4v_1}{\sin^2\theta}\left(\frac{\cos^2\theta}{4} - 1\right) = -\frac{4v_1\cos^2\theta}{4\sin^2\theta} + \frac{4v_1\cos^2\theta}{\sin^2\theta} + \frac{4v_1\sin^2\theta}{\sin^2\theta}$$

$$v_1 = -v_1\frac{\cos^2\theta}{\sin^2\theta} + 4v_1\frac{\cos^2\theta}{\sin^2\theta} + 4v_1$$

$$v_1 + v_1\frac{\cos^2\theta}{\sin^2\theta} = 4v_1 + 4v_1\frac{\cos^2\theta}{\sin^2\theta} \qquad v_1 = 1$$

$$v_1\left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right) = 4v_1\left(1 + \frac{\cos^2\theta}{\sin^2\theta}\right) \qquad \boxed{v_2 = \frac{2 - \cos\theta}{\sin\theta}}$$

$$-\frac{\cos\theta}{2} + 1 = v_2\frac{\sin\theta}{2} \Rightarrow v_2 = -\frac{\cancel{2}\cos\theta}{\cancel{2}\sin\theta} + \frac{2}{\sin\theta}$$

$$v_1 = \sin^2\theta + \cos^2\theta \qquad v_2 = \frac{2 - \cos\theta}{\sin\theta} = \frac{1 - \cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta}$$

$\hat{S}_\theta = \cos\theta\,\hat{S}_z + \sin\theta\,\hat{S}_x \qquad \hat{S}_\theta|\psi\rangle = 1|\psi\rangle$

$|\psi\rangle = \frac{1}{\sqrt{2(1-\cos\theta)}}\begin{pmatrix}\sin\theta\\1-\cos\theta\end{pmatrix}$

$\hat{S}_\theta|\psi\rangle = \left[\cos\theta\left(\frac{1}{2}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\right)+\sin\theta\left(\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\right)\right]\frac{1}{\sqrt{2(1-\cos\theta)}}\begin{pmatrix}\sin\theta\\1-\cos\theta\end{pmatrix}$

$= \frac{\cos\theta}{2\sqrt{2(1-\cos\theta}}\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}\sin\theta\\1-\cos\theta\end{pmatrix} + \frac{\sin\theta}{2\sqrt{2(1-\cos\theta)}}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}\sin\theta\\1-\cos\theta\end{pmatrix}$

$= \frac{\cos\theta}{2\sqrt{2(1-\cos\theta)}}\begin{pmatrix}\sin\theta\\\cos\theta-1\end{pmatrix} + \frac{\sin\theta}{2\sqrt{2(1-\cos\theta)}}\begin{pmatrix}1-\cos\theta\\\sin\theta\end{pmatrix}$

$= \frac{1}{2\sqrt{2(1-\cos\theta)}}\left[\begin{pmatrix}\cancel{\cos\theta\sin\theta}\\\cos^2\theta-\cos\theta\end{pmatrix}+\begin{pmatrix}\sin\theta-\cancel{\sin\theta\cos\theta}\\\sin^2\theta\end{pmatrix}\right]$

$= \frac{1}{2\sqrt{2(1-\cos\theta)}}\begin{pmatrix}\sin\theta\\1-\cos\theta\end{pmatrix} = \frac{1}{2}|\psi\rangle$

$\left[\cos\theta\left(\frac{1}{2}\begin{pmatrix}0&0\\0&-2\end{pmatrix}\right)+\sin\theta\left(\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\right)\right]\begin{pmatrix}v_1\\v_2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$= \frac{1}{2}\cos\theta\begin{pmatrix}0\\-2v_2\end{pmatrix}+\frac{1}{2}\sin\theta\begin{pmatrix}v_2\\v_1\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$

$\Rightarrow \frac{1}{2}\sin\theta\, v_2 = 0\ ;\ v_2 = 0$

$-\cancel{\cos\theta\, v_2} + \frac{1}{2}\sin\theta\, v_1 = 0$

$$\hat{S}_\theta = \cos\theta\, \hat{S}_z + \sin\theta\, \hat{S}_x = \cos\theta \begin{pmatrix} \sin^2\theta + \cos^2\theta & 0 \\ 0 & -\sin^2\theta - \cos^2\theta \end{pmatrix}$$

$$+ \sin\theta \begin{pmatrix} 0 & \sin^2\theta + \cos^2\theta \\ \sin^2\theta + \cos^2\theta & 0 \end{pmatrix}$$

$$\cos\theta \begin{pmatrix} \sin^2\theta + \cos^2\theta - 1 & 0 \\ 0 & -\sin^2\theta - \cos^2\theta - 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} + \sin\theta \begin{pmatrix} 0 & \sin^2\theta + \cos^2\theta \\ \sin^2\theta + \cos^2\theta & 0 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$v_1 = \sin\theta$

$$(\cos\theta \sin^2\theta + \cos\theta \cos^2\theta - \cos\theta) v_1 + 0 + (\sin\theta \sin^2\theta + \sin\theta \cos^2\theta) v_2 = 0$$

$$\cos\theta \sin\theta \sin^2\theta + \cos\theta \sin\theta \cos^2\theta - \cos\theta \sin\theta + (\sin\theta \sin^2\theta + \sin\theta \cos^2\theta) v_2 = 0$$

$$\cos\theta \sin\theta (\sin^2\theta + \cos^2\theta - 1) + \sin\theta\, v_2 = 0$$

$$v_2 \sin\theta = -\cos\theta \sin\theta (1 - \sin^2\theta - \cos^2\theta)$$

$$v_2 = \cos\theta (1 - \sin^2\theta - \cos^2\theta) \quad \text{and} \quad v_1 = \sin\theta$$

$$= \cos\theta - (\cos\theta \sin\theta) \sin\theta - \cos\theta (\cos\theta \cos\theta)$$

$$A^2 \begin{bmatrix} 1 & \frac{1-\cos\theta}{\sin\theta} \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1-\cos\theta}{\sin\theta} \end{bmatrix} = A^2 \left(1 + \left(\frac{1-\cos\theta}{\sin\theta} \right)^2 \right)$$

$$1 = A^2 \left(1 + \frac{1 - 2\cos\theta + \cos^2\theta}{\sin^2\theta} \right) = A^2 \left(\frac{\sin^2\theta + 1 - 2\cos\theta + \cos^2\theta}{\sin^2\theta} \right)$$

("required probability") $\qquad 1 = A^2 \left(\frac{2 - 2\cos\theta}{\sin^2\theta} \right)$

$$A^2 = \left(\frac{\sin^2\theta}{2 - 2\cos\theta} \right) \quad \text{so} \quad A = \frac{\sin\theta}{\sqrt{2 - 2\cos\theta}}$$

$$|\psi\rangle = \frac{\sin\theta}{\sqrt{2 - 2\cos\theta}} \begin{bmatrix} 1 \\ \frac{1-\cos\theta}{\sin\theta} \end{bmatrix} = \boxed{\frac{1}{\sqrt{2(1-\cos\theta)}} \begin{bmatrix} \sin\theta \\ 1 - \cos\theta \end{bmatrix}} \star$$

$\frac{\sin^2\theta}{2(1-\cos\theta)}$

$= -$

$$\star^2 = \frac{1}{2(1-\cos\theta)} \left[\sin^2\theta + (1-\cos\theta)^2 \right], \qquad \frac{1-\cos\theta}{2}$$

$$\Rightarrow \frac{\sin^2\theta}{2(1-\cos\theta)} + \cancel{\frac{(1-\cos\theta)^2}{2(1-\cos\theta)}}$$

$$\frac{\sin^2\theta}{2(1-\cos\theta)} \Rightarrow \frac{\sin^2\theta(1+\cos\theta)}{2(1-\cos\theta)(1+\cos\theta)} = \frac{\sin^2\theta + \sin^2\theta\cos\theta}{2(1-\cos\theta+\cos\theta+\cos^2\theta)}$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}} \Rightarrow \cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{2}$$

$$\frac{\sin^2\theta}{2(1-\cos\theta)} = \frac{1-\cos^2\theta}{2(1-\cos\theta)} = \frac{\cancel{(1-\cos\theta)}(1+\cos\theta)}{2\cancel{(1-\cos\theta)}} = \frac{(1+\cos\theta)}{2}$$

$$\frac{\sin^2\theta}{2(1-\cos\theta)} = \frac{1-\cos^2\theta}{2(1-\cos\theta)} = \frac{\cancel{(1-\cos\theta)}(1+\cos\theta)}{2\cancel{(1-\cos\theta)}} = \frac{(1+\cos\theta)}{2} = \boxed{\cos^2\left(\frac{\theta}{2}\right)}$$

(1.9) Use basis: $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$; $|\uparrow_z\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$|\uparrow\uparrow\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$; $|\uparrow\downarrow\rangle = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}$; $|\downarrow\uparrow\rangle = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}$; $|\downarrow\downarrow\rangle = \begin{pmatrix}0\\0\\1\\0\end{pmatrix}$ (?)

$\hat{S}_z^a \cdot \hat{S}_z^b$ ... $\hat{S}_z^a = \frac{1}{2}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$ $\hat{S}_z^a|\uparrow\uparrow\rangle = \frac{1}{2}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0\\0\\-1\\0\end{pmatrix}$

$$\underline{\hat{S}}^a = \begin{pmatrix}\hat{S}_x^a\\\hat{S}_y^a\\\hat{S}_z^a\end{pmatrix} = \vec{i}\hat{S}_x^a + \vec{j}S_y^a + \vec{k}S_z^a = \vec{i}\,\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix} + \vec{j}\,\frac{1}{2}\begin{pmatrix}0&i\\-i&0\end{pmatrix} + \vec{k}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$|\uparrow_z^a\rangle = \begin{pmatrix}1\\0\end{pmatrix}$; $|\uparrow_z^b\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix} \Rightarrow |\uparrow_z^a\rangle \otimes |\uparrow_z^a\rangle = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

$|\psi\rangle = \begin{pmatrix}a\\b\end{pmatrix} = a|\uparrow_z\rangle + b|\downarrow_z\rangle = a\begin{pmatrix}1\\0\end{pmatrix} + b\begin{pmatrix}0\\1\end{pmatrix}$

vs.

$|\psi\rangle = \begin{pmatrix}a\\b\\c\\d\end{pmatrix} = a|\uparrow\uparrow_z\rangle + b|\uparrow\downarrow\rangle + c|\downarrow\uparrow\rangle + d|\downarrow\downarrow\rangle$

$= a\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + b\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + c\begin{pmatrix}0\\0\\1\\0\end{pmatrix} + d\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$

$I^{(2)}$

$S_z^a = \frac{1}{2}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}$ $-I^{(2)}$ $\frac{1}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}$ $= \begin{pmatrix}1\begin{pmatrix}1&0\\0&1\end{pmatrix} & \begin{matrix}0&0\\0&0\end{matrix}\\ \begin{matrix}0&0\\0&0\end{matrix} & -1\begin{pmatrix}1&0\\0&1\end{pmatrix}\end{pmatrix}$

S_z

$S_z \otimes I = \begin{pmatrix}1&0\\0&-1\end{pmatrix} \otimes \begin{pmatrix}1&0\\0&1\end{pmatrix} = \begin{pmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{pmatrix}$

$S_z^b = \frac{1}{2}\begin{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}&\\&\begin{matrix}1&\\&-1\end{matrix}\end{pmatrix}$

$$\hat{S}_z^a = \hat{S}_z^{(a)} \otimes I^{(b)} = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} = \begin{pmatrix} 1\begin{pmatrix}1&0\\0&1\end{pmatrix} & \begin{matrix}0&0\\0&0\end{matrix}\\ \begin{matrix}0&0\\0&0\end{matrix} & -1\begin{pmatrix}1&0\\0&1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{pmatrix}$$

$$\hat{S}_z^b = I^{(a)} \otimes \hat{S}_z^{(b)} = \begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&-1\end{pmatrix} = \begin{pmatrix} 1\begin{pmatrix}1&0\\0&-1\end{pmatrix} & \begin{matrix}0&0\\0&0\end{matrix}\\ \begin{matrix}0&0\\0&0\end{matrix} & 1\begin{pmatrix}1&0\\0&-1\end{pmatrix}\end{pmatrix} = \begin{pmatrix}1&&&\\&-1&&\\&&1&\\&&&-1\end{pmatrix}$$

$$\hat{S}_x^a = \frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} = \frac{1}{2}\begin{pmatrix} \begin{matrix}0&0\\0&0\end{matrix} & 1\begin{pmatrix}1&0\\0&1\end{pmatrix}\\ 1\begin{pmatrix}1&0\\0&1\end{pmatrix} & \begin{matrix}0&0\\0&0\end{matrix}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}$$

$$\hat{S}_x^b = \begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\frac{1}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1\begin{pmatrix}0&1\\1&0\end{pmatrix} & \begin{matrix}0&0\\0&0\end{matrix}\\ \begin{matrix}0&0\\0&0\end{matrix} & 1\begin{pmatrix}0&1\\1&0\end{pmatrix}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$$

$$\hat{S}_y^a = \frac{1}{2}\begin{pmatrix}0&i\\-i&0\end{pmatrix}\otimes\begin{pmatrix}1&0\\0&1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0&0&i&0\\0&0&0&i\\-i&0&0&0\\0&-i&0&0\end{pmatrix}$$

$$\hat{S}_y^b = \begin{pmatrix}1&0\\0&1\end{pmatrix}\otimes\frac{1}{2}\begin{pmatrix}0&i\\-i&0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}0&i&0&0\\-i&0&0&0\\0&0&0&i\\0&0&-i&0\end{pmatrix}$$

$$\underline{\hat{S}_a} \cdot \underline{\hat{S}_b} = \begin{pmatrix}\hat{S}_x^a & \hat{S}_y^a & \hat{S}_z^a\end{pmatrix}\begin{pmatrix}\hat{S}_x^a\\ \hat{S}_y^a\\ \hat{S}_z^a\end{pmatrix} = (\hat{S}_x^a\hat{S}_x^b) + (\hat{S}_y^a\hat{S}_y^b) + (\hat{S}_z^a\hat{S}_z^b)$$

First term $= \frac{1}{4}\begin{pmatrix}0&0&1&0\\0&0&0&1\\1&0&0&0\\0&1&0&0\end{pmatrix}\cdot\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix} = \frac{1}{4}\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix}$

2nd term $= \frac{1}{4}\begin{pmatrix}0&0&i&0\\0&0&0&i\\-i&0&0&0\\0&-i&0&0\end{pmatrix}\cdot\begin{pmatrix}0&i&0&0\\-i&0&0&0\\0&0&0&i\\0&0&-i&0\end{pmatrix} = \frac{1}{4}\begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&1&0&0\\-1&0&0&0\end{pmatrix}$

3rd term $= \frac{1}{4}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}\cdot\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix} = \frac{1}{4}\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}$

$$\boxed{\underline{\hat{S}_a} \cdot \underline{\hat{S}_b} = \frac{1}{4}\left[\begin{pmatrix}0&0&0&1\\0&0&1&0\\0&1&0&0\\1&0&0&0\end{pmatrix} + \begin{pmatrix}0&0&0&-1\\0&0&1&0\\0&1&0&0\\-1&0&0&0\end{pmatrix} + \begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix}\right] = \frac{1}{4}\begin{pmatrix}1&0&0&0\\0&-1&2&0\\0&2&-1&0\\0&0&0&1\end{pmatrix}}$$

$$A = \hat{S}_a \cdot \hat{S}_b = \frac{1}{4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}; \quad \det(A - \lambda I) = \emptyset$$

$$\det\begin{pmatrix} 1/4-\lambda & 0 & 0 & 0 \\ 0 & -1/4-\lambda & 2 & 0 \\ 0 & 2 & -1/4-\lambda & 0 \\ 0 & 0 & 0 & 1/4-\lambda \end{pmatrix} = \lambda^4 - \frac{33\lambda^2}{8} + 2\lambda - \frac{63}{256}$$

↳ solve w/ Mathematica gives $\lambda_1 = \frac{1}{4}, \lambda_2 = \frac{1}{4}, \lambda_3 = 2\frac{1}{4}, \lambda_4 = -1\frac{3}{4}$

... I know the procedure, so not worth my time to do by hand.

(1.11) $\hat{S}_{\theta,\varphi} = \sin\theta\cos\varphi\,\hat{S}_x + \sin\theta\sin\varphi\,\hat{S}_y + \cos\theta\,\hat{S}_z$

$$\hat{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{S}_y = \frac{1}{2}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, \quad \hat{S}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\hat{S}_{\theta,\varphi} = \frac{1}{2}\left[\begin{pmatrix} \emptyset & \sin\theta\cos\varphi \\ \sin\theta\cos\varphi & \emptyset \end{pmatrix} + \begin{pmatrix} \emptyset & i\sin\theta\sin\varphi \\ -i\sin\theta\sin\varphi & \emptyset \end{pmatrix} + \begin{pmatrix} \cos\theta & \emptyset \\ \emptyset & -\cos\theta \end{pmatrix}\right]$$

$$= \frac{1}{2}\begin{pmatrix} \cos\theta & \sin\theta\cos\varphi + i\sin\theta\sin\varphi \\ \sin\theta\cos\varphi - i\sin\theta\sin\varphi & -\cos\theta \end{pmatrix}$$

$$\det\left(\begin{pmatrix} \frac{1}{2}\cos\theta - \lambda & \frac{1}{2}\sin\theta\cos\varphi + i\sin\theta\sin\varphi \\ \frac{1}{2}\sin\theta\cos\varphi - i\ldots & -\frac{1}{2}\cos\theta - \lambda \end{pmatrix}\right) = \emptyset$$

$$\left(\frac{1}{2}\cos\theta - \lambda\right)\left(-\frac{1}{2}\cos\theta - \lambda\right) - \left(\frac{1}{2}\sin\theta\cos\varphi + i\sin\theta\sin\varphi\right)\left(\frac{1}{2}\sin\theta\cos\varphi - i\sin\theta\sin\varphi\right)$$

$$= -\frac{1}{4}\cos^2\theta - \cancel{\frac{\lambda}{2}\cos\theta} + \cancel{\frac{\lambda}{2}\cos\theta} + \lambda^2 - \frac{1}{4}\sin^2\theta\cos^2\varphi + \cancel{\frac{i}{2}\sin^2\theta\sin\varphi\cos\varphi}$$

$$- \cancel{\frac{i}{2}\sin^2\theta\sin\varphi\cos\varphi} - \sin^2\theta\sin^2\varphi$$

$$= -\frac{1}{4} + \lambda^2 + \frac{1}{4}\sin^2\theta - \frac{1}{4}\sin^2\theta\cos^2\varphi - \sin^2\theta(1 - \cos^2\varphi)$$

$$= -\frac{1}{4} + \lambda^2 + \frac{1}{4}\sin^2\theta - \frac{1}{4}\sin^2\theta\cos^2\varphi - \sin^2\theta + \sin^2\theta\cos^2\varphi$$

$$= -\frac{1}{4} + \lambda^2 - \frac{3}{4}\sin^2\theta + \frac{3}{4}\sin^2\theta\cos^2\varphi - \ldots$$ algebra...

(1.13) Show that $\frac{d}{dt}\langle \hat{\underline{S}} \rangle = \frac{1}{i\hbar}\langle [\hat{\underline{S}}, \mathcal{H}] \rangle = -\frac{g\mu_B}{\hbar}\langle \hat{S} \rangle \times \hat{B}$

using $\hat{\mathcal{H}} = g\mu_B \hat{\underline{B}} \cdot \hat{\underline{S}}$ + $\frac{d\langle \hat{A} \rangle}{dt} = \frac{1}{i\hbar}\langle [\hat{A}, \mathcal{H}] \rangle$

$\frac{d}{dt}\langle \hat{\underline{S}} \rangle = \frac{1}{i\hbar}\langle [\hat{S}, \hat{\mathcal{H}}] \rangle = \frac{1}{i\hbar}(\hat{S}\hat{\mathcal{H}} - \hat{\mathcal{H}}\hat{S})$

$= \frac{1}{i\hbar}\left[\hat{S}(g\mu_B \hat{\underline{B}} \cdot \hat{S}) - (g\mu_B \hat{B} \cdot \hat{S})\hat{S}\right]$

$= \frac{g\mu_B}{i\hbar}[\hat{S} \cdot \hat{B} \cdot \hat{S} - \hat{B} \cdot \hat{S} \cdot \hat{S}] = \frac{g\mu_B}{i\hbar}[(\hat{S} \cdot \hat{B} - \hat{B} \cdot \hat{S}) \cdot \hat{S}]$

kpHam[korder, input, "Method"]

can be iterative simplification or direct product decomp. default is ISA which is faster

format of "Association"

Three necessary inputs:

1.) The rotation part of Q
2.) The (co)representation matrix of Q
3.) whether Q is a unitary or anti-unitary operator

Q is some symmetry operation?

Name it, could be say "C3"

input = <|
"Unitary" -> <|Q1 -> {D(Q1), -R, $\vec{k}$}, ...|>,
"Antiunitary" -> <|Q2 -> {D(Q2), -K, $\vec{k}$}, ...|>
|>

can get this matrix from command

Does # of bands come from order of expansion?

If I know the space group number of our ML WTe_2 cell, I can use the interface Rep[] command (along with the k-point around which I'm expanding)

reps is an integer or list of integers which represents the serial # of irreducible (co)representations

Each column of Ham draws a given band!