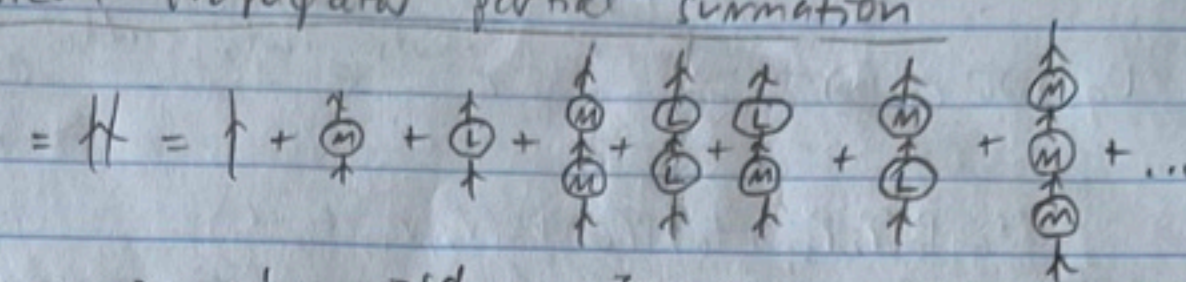
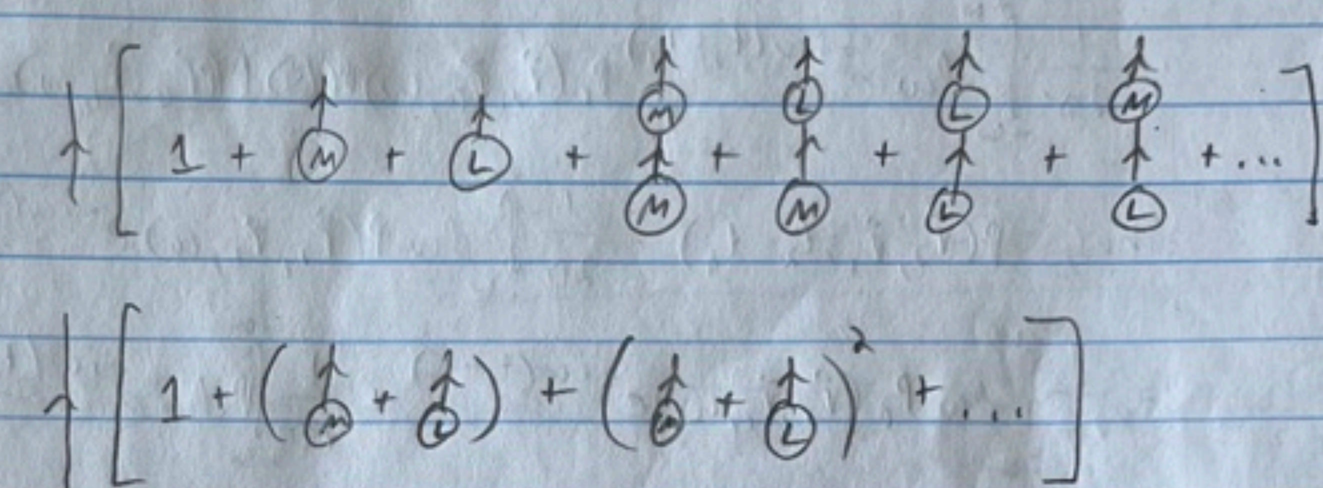
Chp 2: Classical Propagator partial summation

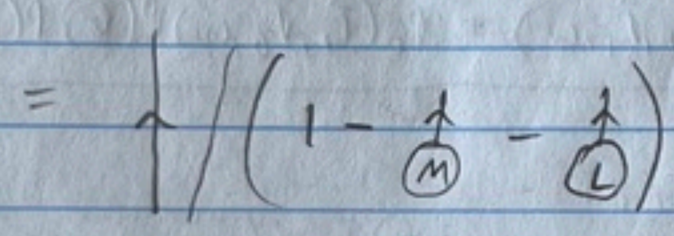
2.1) $P(\vec{r}_2, \vec{r}_1) =$ 

In 1st order: $2 = 2^1$ $\quad$ 3rd $\rightarrow 2^3 = 8$

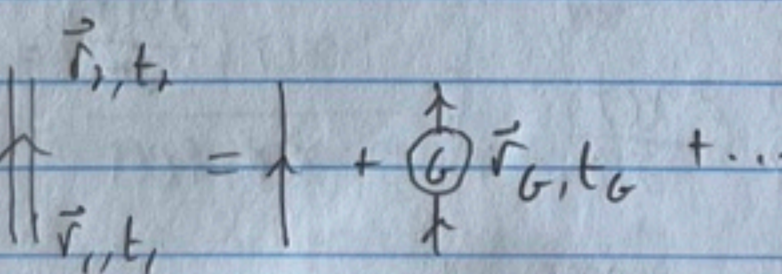
2nd: $4 = 2^2$ $\quad$ nth $\rightarrow \boxed{2^n}$

2.2) $$P(\vec{r}_2, \vec{r}_1) = P_0(\vec{r}_2, \vec{r}_1) + P_0(\vec{r}_2, \vec{r}_M) P(M) P(\vec{r}_M, \vec{r}_1) + P_0(\vec{r}_2, \vec{r}_L) P(L) P(\vec{r}_L, \vec{r}_1) + \dots + P_0(\vec{r}_2, \vec{r}_M) P(M) P_0(\vec{r}_M, \vec{r}_L) P(L) P(\vec{r}_L, \vec{r}_1) + \dots$$

2.3) $P(\vec{r}_2, \vec{r}_1) =$ 

 $$= \frac{c}{c(c^{-1} - P(M) - P(L))} = \boxed{\frac{1}{c^{-1} - P(M) - P(L)}}$$

2.4) $$\boxed{P(\vec{r}_2, \vec{r}_1) = \frac{1}{c^{-1} - \sum_A P(A)}}$$

Eq. 2.20) 

$$P(\vec{r}_2, \vec{r}_1, t_2 - t_1) = P_0(\vec{r}_2, \vec{r}_1, t_2 - t_1) + \int_{t_1}^{t_2} dt_G \, P_0(\vec{r}_2, \vec{r}_G, t_2 - t_G) P(G) P_0(\vec{r}_G, \vec{r}_1, t_G - t_1) + \dots$$

Term 1: $$P_0(\vec{r}_2, \vec{r}_1, t_2 - t_1) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} d\omega \, e^{-i\omega(t_2 - t_1)} P_0(\vec{r}_2, \vec{r}_1, \omega)$$

Term 2 on next page...

Eq. 2.20 (cont.) $e^{-i\omega t_2} e^{i\omega t_G} e^{-i\omega' t_G} e^{i\omega' t_1}$

$\hookrightarrow = e^{i(\omega-\omega')t_G} e^{-i\omega t_2} e^{i\omega' t_1}$

$$\int_{t_1}^{t_2} dt_G \, P_0(\vec{r}_2,\vec{r}_G, t_2-t_G) P(G) P_0(\vec{r}_G, r_1, t_G - t_1)$$

$$= \int_{t_1}^{t_2} dt_G \left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega \, e^{-i\omega(t_2-t_G)} P_0(\vec{r}_2,\vec{r}_G,\omega)\right] \times P(G) \times \left[\frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega' e^{-i\omega'(t_G-t_1)} P_0(\vec{r}_G,\vec{r}_1,\omega')\right]$$

$2\pi\delta(\omega-\omega')$

$$= \left(\frac{1}{2\pi}\right)^2 \int_{t_1}^{t_2} dt_G \, e^{i(\omega-\omega')t_G} \int_{-\infty}^{+\infty} d\omega \int_{+\infty}^{+\infty} d\omega' \, e^{-i(\omega t_2 - \omega' t_1)} P_0(\vec{r}_2,\vec{r}_G,\omega) P(G) P_0(\vec{r}_G,\vec{r}_1,\omega')$$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega \, e^{-i\omega(t_2-t_1)} P_0(\vec{r}_2,\vec{r}_G,\omega) P(G) P(\vec{r}_G,\vec{r}_1,\omega)$$

Thus, $P(\vec{r}_2,\vec{r}_1,t_2-t_1) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega \, P(\vec{r}_2,\vec{r}_1,\omega) e^{-i\omega(t_2-t_1)}$

$$= \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega \, P_0(\vec{r}_2,\vec{r}_1,\omega) e^{-i\omega(t_2-t_1)} + \frac{1}{2\pi}\int_{-\infty}^{+\infty} d\omega \, e^{-i\omega(t_2-t_1)} P_0(\vec{r}_2,\vec{r}_G,\omega) P(G) P_0(\vec{r}_G,\vec{r}_1,\omega)$$

and $\boxed{P(\vec{r}_2,\vec{r}_1,\omega) = P_0(\vec{r}_2,\vec{r}_1,\omega) + P_0(\vec{r}_2,\vec{r}_G,\omega)P(G)P(\vec{r}_G,\vec{r}_1,\omega) + \ldots}$

Eqn. 3.6) $\Psi(\vec{r},t_1) = \varphi_{k_1}(\vec{r})$

Solve $i\frac{\partial}{\partial t}\Psi(\vec{r},t) = \hat{H}\Psi(\vec{r},t)$ for $\Psi(\vec{r},t)$ + eigenvalues with $\hat{H} = -\frac{\nabla_r^2}{2m}$ $\Rightarrow$ Assume $\Psi(\vec{r},t) = \chi(r) f(t)$

$$i\frac{\partial}{\partial t}[\chi(\vec{r})f(t)] = i\chi(\vec{r})\frac{\partial f(t)}{\partial t}$$

$$-\frac{\nabla_r^2}{2m}\chi(r)f(t) = \frac{f(t)}{2m}\frac{\partial^2\chi(\vec{r})}{\partial \vec{r}^2}$$

$$\frac{i\chi(\vec{r})\frac{df(t)}{dt}}{\chi(\vec{r})f(t)} = \frac{\frac{d^2\chi(\vec{r})}{d\vec{r}^2}\frac{f(t)}{2m}}{\chi(\vec{r})f(t)}$$

$$\hookrightarrow i\frac{df(t)}{dt}\frac{1}{f(t)} = -\frac{1}{\chi(\vec{r})}\frac{d^2\chi(\vec{r})}{d\vec{r}^2}\frac{1}{2m}$$

$$i\frac{df(t)}{dt}\frac{1}{f(t)} = E \rightarrow \frac{df(t)}{dt} = -if(t)E \rightarrow e^{-iEt} f(t') \quad \uparrow e^{-iE(t-t')}$$

$$\hookrightarrow \frac{1}{f(t)}df = -iE\,dt \quad \text{and} \quad \int \frac{1}{f}df = \ln f + C$$

$$\ln f = -iEt + (D-C) \rightarrow f = e^{-iEt} \underbrace{e^{(D-C)}}_{f(0)} \quad \int -iE\,dt = -iEt + D$$